Elliptic regularity estimates with optimized constants and applications

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(Joint work with Boyan Sirakov) We revisit the classical theory of linear second-order uniformly elliptic equations in divergence form and prove versions of the generalized maximum principle, the $C^{1,\alpha}$ estimate, the Hopf-Oleinik lemma, the boundary weak Harnack inequality and the differential Harnack inequality, in which the constant is optimized with respect to the norms of the coefficients of the operator and the size of the domain.

The optimal constants turn out to have an exponential dependence in these quantities. Our estimates are complemented by counterexamples which show their optimality. We then give applications to the Landis conjecture and to spectral estimates.