The steady Navier-Stokes equations in a system of unbounded channels with sources and sinks

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Abstract

The steady motion of a viscous incompressible fluid in a junction of unbounded channels with sources and sinks is modeled through the Navier-Stokes equations under inhomogeneous Dirichlet boundary conditions. Under a general outflow constraint, we prove the existence of a solution with a uniformly bounded Dirichlet integral in every compact subset. In a simplified framework, this problem was proposed by Jean Leray to Olga Ladyzhenskaya during his visit to Leningrad in 1958.

The main novelties of our approach are the construction of a flux carrier satisfying a uniform Leray-Hopf inequality in rectangular sections and the proof of some properties of weak solutions to the stationary Euler equations in bounded planar domains, such as the regularity of the extension to the whole plane, of the related Bernoulli pressure and of the stream function. This regularity is used to obtain local Morse-Sard-type information and to generate a solution through the *invading domains* procedure. For small data of the problem, we also prove unique solvability and attainability of Couette-Poiseuille flows at infinity.

This is a joint work with Mikhail Korobkov (Shanghai), Xiao Ren (Beijing), Gianmarco Sperone (Santiago de Chile).