L^∞ COMPACTNESS OF SOLUTIONS OF LERAY-LIONS QUASILINEAR PROBLEMS

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We present the **joint work** with **M. C. M. Rezende** and **E. A. B. Silva** (Universidade de Brasília). For a (**not necessarily smooth**) bounded domain Ω of $\mathbb{R}^N$, $N \geq 2$ and a Carathéodory vector valued function $a : \Omega \times \mathbb{R}^N \to \mathbb{R}^N$, we study the compactness of the inverse of the Leray-Lions operator $A(u) = -\text{div}(a(x, \nabla u))$, $u \in W_0^{1,p}(\Omega)$, $1 < p \leq N$. Denoting by p^* the Sobolev exponent, in the main result it is proved the compactness of $A^{-1} : L^\sigma(\Omega) \to L^\infty(\Omega)$ if $2N/(N+2) < p \leq N$, provided $\sigma > \max\{N/p, (p^*/2)'\}$. Also, for $\sigma > p_* := (p^*)'$, the operators $A^{-1} : L^\sigma(\Omega) \to W_0^{1,p}(\Omega)$ and $A^{-1} : L^\sigma(\Omega) \to L^q(\Omega)$ are compact for every $1 \leq q < [(p-1)\sigma^*]^*$. In contrast, it is also established that $A^{-1} : L^{p_*}(\Omega) \longrightarrow L^{p^*}(\Omega)$ and $A^{-1} : L^{p_*}(\Omega) \to W_0^{1,p}(\Omega)$ are not compact. The compactness of the set of solutions for the more general operator $A(u) = -\text{div}(a(x, u, \nabla u))$ is also studied. As an application we improve previous results on the existence and multiplicity of solutions for a class of problems involving the p-Laplacian operator under a local Landesman-Lazer condition for arbitrary bounded domains.