NONLINEAR ELLIPTIC PDE IN HAUTS-DE-FRANCE (4th edition)

INVITED SPEAKERS :

- Carlo Alberto Antonini (Università di Parma)
- David Arcoya (Universidad de Granada)
- Thomas Bartsch (Universität Giessen)
- Denis Bonheure (Université Libre de Bruxelles)
- Silvia Cingolani (Università degli Studi di Bari Aldo Moro)
- Giulio Ciraolo (Università degli Studi di Milano La Statale)
- André de Laire (Université de Lille)
- Simone Dovetta (Politecnico di Torino)
- Antonio J. Fernández (Universidad Autónoma de Madrid)
- Damien Galant (Université de Mons)
- Filippo Gazzola (Politecnico di Milano)
- François Hamel (Université d'Aix-Marseille)
- Louis Jeanjean (Université de Franche-Comté)
- François Murat (Sorbonne Université)
- Rosa Pardo (Universidad Complutense de Madrid)
- Angela Pistoia (Università di Roma Sapienza)
- Enrico Serra (Politecnico di Torino)
- Philippe Souplet (Université Sorbonne Paris Nord)
- Guido Sweers (Universität zu Köln)
- Christophe Troestler (Université de Mons)

16 > 19 JUNE 2025

Université du Littoral Côte d'Opale
CALAIS - FRANCE

ORGANIZING COMMITTEE

- Mabel Cuesta (LMPA)
- Colette De Coster (CERAMATHS)
- Alberto Farina (LAMFA)
- Gaëlle Mailly (LMPA)

PROGRAM, INFORMATIONS AND REGISTRATION
https://4th-nlepde-hf.sciencesconf.org

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1 Schedule

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11:30 - 12:15	**Rosa Pardo**
12:20 - 14:00	*Lunch*
	Chairman: G. Sweers
14:00 - 14:45	**Damien Galant**
14:50 - 15:35	**Christophe Troestler**
15:40 - 16:10	*Coffee Break*
	Chairman: D. Arcoya
16:10 - 16:55	**François Hamel**
17:00 - 17:45	**Enrico Serra**

1.2 Tuesday 17/06

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Visit/Tour	

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	Chairman: R. Pardo
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1.4 Thursday 19/06

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9:15 - 10:00	**Philippe Souplet**
10:05 - 10:50	**Carlo Alberto Antonini**
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	Chairman: C. De Coster
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12:15 - 14:00	*Lunch*

2 Abstracts

Regularity results and maximum principles for quasilinear operators of mixed local-nonlocal type

Carlo Alberto Antonini

In this talk, we will deal with mixed local-nonlocal quasilinear operators, modeled upon the sum of a p-Laplacian and a fractional (s,q)-Laplace operator, i.e., $-\Delta_p u + (-\Delta_q)^s u$.

We will review some recent results concerning local and global regularity of their solutions, and we will address the validity of maximum principles and Hopf Lemma for such operators.

Based on a joint work with M. Cozzi.

L^∞ compactness of solutions of Leray-Lions quasilinear problems

David Arcoya

We present the joint work with M. C. M. Rezende and E. A. B. Silva (Universidade de Brasília). For a (**not necessarily smooth**) bounded domain Ω of $\mathbb{R}^N$, $N \geqslant 2$ and a Carathéodory vector valued function $a : \Omega \times \mathbb{R}^N \to \mathbb{R}^N$, we study the compactness of the inverse of the Leray-Lions operator $A(u) = -\mathrm{div}(a(x, \nabla u))$, $u \in W_0^{1,p}(\Omega)$, $1 < p \leqslant N$. Denoting by p^* the Sobolev exponent, in the main result it is proved the compactness of $A^{-1} : L^\sigma(\Omega) \to L^\infty(\Omega)$ if $2N/(N+2) < p \leqslant N$, provided $\sigma > \max\{N/p, (p^*/2)'\}$. Also, for $\sigma > p_* := (p^*)'$, the operators $A^{-1} : L^\sigma(\Omega) \to W_0^{1,p}(\Omega)$ and $A^{-1} : L^\sigma(\Omega) \to L^q(\Omega)$ are compact for every $1 \leqslant q < [(p-1)\sigma^*]^*$. In contrast, it is also established that $A^{-1} : L^{p_*}(\Omega) \longrightarrow L^{p^*}(\Omega)$ and $A^{-1} : L^{p_*}(\Omega) \to W_0^{1,p}(\Omega)$ are not compact. The compactness of the set of solutions for the more general operator $A(u) = -\mathrm{div}(a(x, u, \nabla u))$ is also studied. As an application we improve previous results on the existence and multiplicity of solutions for a class of problems involving the p-Laplacian operator under a local Landesman-Lazer condition for arbitrary bounded domains.

Normalized solutions of Schrödinger equations on domains

Thomas Bartsch

The existence of solutions $(\lambda, u) \in \mathbb{R} \times H_0^1(\Omega)$ of nonlinear Schrödinger equations like

$$-\Delta u + V(x)u + \lambda u = f(u) \qquad \text{in } \Omega \subseteq \mathbb{R}^N \tag{1}$$

with prescribed L^2-norm

$$\int_\Omega u^2 = a \tag{2}$$

has found considerable interest in the last decade. If $\Omega = \mathbb{R}^N$ and V is constant then the scaling $s * u(x) = s^{N/2}u(sx)$ plays an important role in proving the Palais-Smale condition for the associated functional on the L^2-sphere. We present recent results on the existence of solutions of (1)-(2) on bounded domains where this scaling cannot be used.

The talk is based on work with Shijie Qi and Wenming Zou.

Particles almost in contact : the general method of reduced functionals in 3D

Denis Bonheure

The formal analysis of the forces and torques on two moving solid particles suspended in a laminar flow and almost in contact with each other (or on a particle almost in contact with the wall of a container) traces back at least to Brenner and Cox in the late 1960's by using lubrication theory. While the stream function is defined up to a constant in 2D, the vector potential in 3D is defined up to a gradient and the choice of a gauge. I will show that by choosing an ad-hoc gauge, one can find the optimal potential by solving the dual formulation of a resulting Euler-Lagrange equation. This allows to compute (and fully justify) the asymptotic expression of the Stokes resistance matrix.

The construction is fully variational while the sharp asymptotics are basically based on estimates for a weighted elliptic operator in divergence form. I will start the talk by explaining the method on the easier problem of estimating the relative capacity of sets close to contact and showing a link with a missing weighted Hardy inequality.

The talk is based on a joint project with E. Bocchi (Pol. Milano) and M. Hillairet (Univ. Montpellier).

On N-euclidean logarithmic Moser-Trudinger -Onofri inequality and some geometrical variants

Silvia Cingolani

In my talk I will extend the N-dimensional Euclidean Onofri inequality proved by Del Pino and Dolbeault for smooth compactly supported functions in $\mathbb{R}^N$, with $N \geqslant 2$, to a suitable weighted Sobolev space. I will show that in any dimension $N \geqslant 2$, the Euclidean Onofri inequality is equivalent to the logarithmic Moser-Trudinger inequality with sharp constant proved by Carleson and Chang for balls in $\mathbb{R}^N$.

This is a joint paper with N. Borgia and G. Mancini.

Classification results, rigidity theorems and semilinear pdes on Riemannian manifolds: a P-Function approach

Giulio Ciraolo

We consider solutions to critical and sub-critical semilinear elliptic equations on complete, noncompact Riemannian manifolds and study their classification as well as the effect of their presence on the underlying manifold. When the Ricci curvature is non-negative, we prove both the classification of positive solutions to the critical equation and the rigidity for the ambient manifold. The same results are established for solutions to the Liouville equation on Riemannian surfaces. Our results are obtained via an appropriate P-function whose constancy implies the classification of both the solutions and the underlying manifold. The analysis carried out on the P-function also makes it possible to classify non-negative solutions for subcritical equations on manifolds enjoying a Sobolev inequality and satisfying an integrability condition on the negative part of the Ricci curvature.

This is a joint work with Alberto Farina and Camilla Chiara Polvara.

Minimizing travelling waves for the Gross-Pitaevskii equation on the 2D strip

André DE LAIRE

We investigate the two-dimensional defocusing nonlinear cubic Schrö -dinger (Gross-Pitaevskii) equation with nonzero conditions at infinity, on a strip, i.e. on an infinite channel of finite transverse width. We establish the existence of traveling waves that minimize the Ginzburg-Landau energy at fixed momentum. We establish a sharp bifurcation from planar to multidimensional behavior. Precisely, we show that there exists a threshold value for the transverse width below which minimizers are the one-dimensional dark solitons (planar solitons), and above which they are genuinely two-dimensional dark solitons. The existence of minimizers follows from a compactness argument based on a novel periodic symmetrization.

This is a joint work with Didier Smets and Philippe Gravejat.

Non-uniqueness of normalized ground states for nonlinear Schrödinger equations

Simone DOVETTA

The talk discusses general non-uniqueness results for normalized ground states of nonlinear Schrödinger equations with power nonlinearity. Basically, we show that, whenever in the L^2-subcritical regime ground states exist at every mass, for nonlinearity powers close to the L^2-critical exponent there is at least one value of the mass for which ground states are non-unique. As a consequence, we also show that, whenever such non-uniqueness occurs, there exist action ground states that are not normalized ground states. These results have been obtained both when the problem is set on metric graphs (compact and non-compact) and when it is posed on bounded domains with homogeneous Neumann boundary conditions.

Smooth nonradial stationary 2d Euler flows with compact support

Antonio J. Fernández

In this talk we will show how to construct nonradial classical solutions to the 2d incompressible Euler equations. More precisely, for any positive integer k, we will see how to construct compactly supported stationary Euler flows of class $C^k(\mathbb{R}^2)$ which are not locally radial.

The talk is based on a joint work with Alberto Enciso (Madrid) and David Ruiz (Granada).

Qualitative properties of solutions of the nonlinear Schrödinger equation on metric graphs

Damien Galant

In this talk, we consider the nonlinear Schrödinger equation set on compact metric graphs. More precisely, we study qualitative properties of solutions (uniqueness, symmetries, nodal zones...). As this is a very delicate question in general, we focus on the "near-linear" regime. In this case, one can perform a Lyapunov-Schmidt reduction which leads us to study a "reduced problem" set on (finite dimensional) eigenspaces of the Laplacian on the graph. We will show that there are new phenomenon specific to graphs that we have to take care about. We then apply these results to the detailed study of solutions on some specific compact graphs to illustrate a varied behavior.

This is joint work with Colette De Coster (CERAMATHS/DMATHS) and Christophe Troestler (UMONS).

The steady Navier-Stokes equations in a system of unbounded channels with sources and sinks

Filippo Gazzola

The steady motion of a viscous incompressible fluid in a junction of unbounded channels with sources and sinks is modeled through the Navier-Stokes equations under inhomogeneous Dirichlet boundary conditions. Under a general outflow constraint, we prove the existence of a solution with a uniformly bounded Dirichlet integral in every compact subset. In a simplified framework, this problem was proposed

by Jean Leray to Olga Ladyzhenskaya during his visit to Leningrad in 1958.

The main novelties of our approach are the construction of a flux carrier satisfying a uniform Leray-Hopf inequality in rectangular sections and the proof of some properties of weak solutions to the stationary Euler equations in bounded planar domains, such as the regularity of the extension to the whole plane, of the related Bernoulli pressure and of the stream function. This regularity is used to obtain local Morse-Sard-type information and to generate a solution through the *invading domains* procedure. For small data of the problem, we also prove unique solvability and attainability of Couette-Poiseuille flows at infinity.

This is a joint work with Mikhail Korobkov (Shanghai), Xiao Ren (Beijing), Gianmarco Sperone (Santiago de Chile).

Log-concavity of solutions of semi-linear elliptic equations in convex domains

François Hamel

In a bounded convex domain, a function which is positive in the interior of the domain and which vanishes on the boundary is called quasi-concave if all its upper level sets are convex. Positive solutions of many classes of semi-linear elliptic equations satisfying Dirichlet boundary conditions are known to be quasi-concave in convex domains, though counter-examples are also known. The principal eigenfunctions of the Dirichlet Laplacian in convex domains are known to be quasi-concave, and even log-concave, that is, their logarithm is concave. In this talk, I will review some of these classical results and report on recent results on log-concavity of solutions beyond the linear case, based on quantified versions of the anti-maximum principle.

The talk is based on joint works with Nikolai Nadirashvili.

A Multiplicity result for a mass supercritical NLS with a partial confinement

Louis Jeanjean

In this talk we consider the existence of solutions for the NLS with partial confinement and mass supercritical nonlinearity,

$$-\Delta u + (x_1^2 + x_2^2)u + \lambda u = |u|^{p-1}u, \quad x \in \mathbb{R}^3. \tag{1}$$

We are interested in prescribed L^2 norm solutions. It corresponds to the search for constrained critical points of the associated *energy* functional on the constraint given by the prescribed L^2 norm.

The existence, for small values of the L^2 norm, of a first solution, characterized as a local minimum of the *energy* functional on the constraint and being an energy ground state was already known. We now show the existence of a second positive solution, at a mountain pass level. Our solution is obtained as a limit of a sequence of solutions of corresponding problems in bounded domains of $\mathbb{R}^3$. The symmetry of the solutions in bounded domains is central to the convergence process.

This is a joint work with Linjie Song.

Unexpected results for a one-dimensional singular elliptic problem

François Murat

In this lecture, I will present results obtained in collaboration with Daniela Giachetti (Rome), Pedro J. Martínez Aparicio (Almería), and Francesco Petitta (Rome), for the one-dimensional singular boundary value problem which is formally written as

$$-\frac{d}{dx}\left(a(x)\frac{du(x)}{dx}\right) = -\frac{dg(x)}{dx} - \frac{d\varphi(u(x))}{dx} \text{ in } (0,L), \quad u(0) = u(L) = 0,$$

where the coefficient $a(x) \in L^\infty(0,L)$ is bounded from below by a positive constant and where in the right-hand side the function $g(x)$ belongs to $L^2(0,L)$, while the model for the singular nonlinear function φ is given by $\varphi(s) = \dfrac{1}{|s|^\gamma}$ with $\gamma > 0$.

This singular problem presents a number of unexpected phenomena: non-existence of solutions under certain assumptions, existence of an infinite number of solutions under other assumptions, and non-continuity of the solutions with respect to the approximations of the data.

Elliptic Systems with Superlinear Terms on the Critical Hyperbola

Rosa Pardo

We focus on semilinear elliptic systems involving a nonlinearity of slightly subcritical nature understood in a generalized sense. For this problem, standard compact embeddings cannot be used to guarantee the existence of solutions as in the case of power-type nonlinearities. Instead, we use the dual method on Orlicz spaces. Roughly speaking, this method consists in taking the inverse of the Laplace operator, and defining the inverse of the nonlinearities. We state sufficient conditions guaranteeing the Palais-Smale condition, showing that our problem possesses a mountain pass type solution.

This is a joint work with Mabel Cuesta and Angela Pistoia, see [1].

References

[1] M. Cuesta, R. Pardo, and A. Pistoia. Positive solutions of elliptic systems with superlinear terms on the critical hyperbola. *Milan J. Math.*, 92(2):439–472, 2024.

Solutions to elliptic systems in a competitive regime

Angela Pistoia

I will present some old and new results concerning existence of positive solutions to a class of systems of PDE's arising in the study of Bose-Einstein condensates in the whole euclidean space in presence of a competitive regime.

An action approach to nodal and least energy normalized solutions for nonlinear Schrödinger equations

Enrico Serra

We develop a new approach to the investigation of normalized solutions for nonlinear Schrödinger equations based on the analysis of the masses of ground states of the corresponding action functional. Our first result is a complete characterization of the masses of action ground states, obtained via a Darboux-type property for the derivative of the action ground state level. We then exploit this result to tackle normalized solutions with a twofold perspective. First, we

prove existence of normalized nodal solutions for every mass in the L^2-subcritical regime, and for a whole interval of masses in the L^2-critical and supercritical cases. Then, we show when least energy normalized solutions/least energy normalized nodal solutions are action ground states/nodal action ground states.

Elliptic regularity estimates with optimized constants and applications

Philippe Souplet

(Joint work with Boyan Sirakov) We revisit the classical theory of linear second-order uniformly elliptic equations in divergence form and prove versions of the generalized maximum principle, the $C^{1,\alpha}$ estimate, the Hopf-Oleinik lemma, the boundary weak Harnack inequality and the differential Harnack inequality, in which the constant is optimized with respect to the norms of the coefficients of the operator and the size of the domain.

The optimal constants turn out to have an exponential dependence in these quantities. Our estimates are complemented by counterexamples which show their optimality. We then give applications to the Landis conjecture and to spectral estimates.

Duffin's superbiharmonic counterexample revisited

Guido Sweers

Duffin in 1946 constructed a superbiharmonic function u, i.e. $\Delta^2 u \geqslant 0$, on the infinite strip $S := \mathbb{R}\times(-1,1)$ satisfying zero Dirichlet boundary conditions $u = \partial_\nu u = 0$ on ∂S, and showed that u changes sign. It was a kind of first counterexample for the Boggio-Hadamard conjecture* from the beginning of the 20th century. We will recall his counterexample, go somewhat further and may find some nice implications.

*Superbiharmonic functions on (convex) domains that satisfy zero Dirichlet boundary conditions are nonnegative.

Exact multiplicity of positive solutions to superlinear elliptic problems with a sign-changing weight

Christophe Troestler

This talk will be concerned with the number of positive solutions to the boundary value problem

$$\begin{cases} -u'' = \big(h^+(t) - \mu h^-(t)\big) g(u), \\ u(a) = 0 = u(b), \end{cases}$$

where $h^{\pm}(t) := \max\{\pm h(t), 0\}$ and g is a superlinear function. The function $h \in L^1([a,b])$ will be assumed to have m intervals where it is positive separated by intervals where it is negative. Under some assumption on the shape of the positive "bumps", we will establish that this problem possesses *exactly* $2^m - 1$ positive solutions whenever μ is large enough. Moreover, these solutions are non-degenerate.

This is a joint work with Guglielmo Feltrin.

3 Participants

- Mohammad Akil, Université Polytechnique Hauts-de-France (Valenciennes, France).
- Carlo Alberto Antonini, Università di Parma (Parma, Italy).
- David Arcoya, Universidad de Granada (Granada, Spain).
- Lara Balbaaki, Technical University of Denmark (Kgs. Lyngbyn, Denmark).
- Thomas Bartsch, Universität Giessen (Giessen, Germany).
- Antoine Benoit, Université du Littoral Côte d'Opale (Calais, France).
- Nicolas Beuvin, Université de Picardie Jules Vernes (Amiens, France).
- Ralf Blossey, Université de Lille, CNRS (Lille, France).
- Denis Bonheure, Université Libre de Bruxelles (Bruxelles, Belgium).
- Natalino Borgia, Università degli Studi di Bari Aldo Moro (Bari, Italy).
- Alessandro Cannone, Università degli Studi di Bari Aldo Moro (Bari, Italy).
- Pablo Carrillo Martinez, Université de Bourgogne Franche-Comté (Besançon, France).
- Loth Damagui Chabi, Université Sorbone Paris Nord (Paris, France).
- Silvia Cingolani, Università degli Studi di Bari Aldo Moro (Bari, Italy).
- Giulio Ciraolo, Università degli Studi di Milano La Statale (Milan, Italy).
- Mabel Cuesta, Université du Littoral Côte d'Opale (Calais, France).
- Colette De Coster, Université Polytechnique Hauts-de-France (Valenciennes, France).
- André de Laire, Université de Lille (Lille, France).
- Simone Dovetta, Politecnico di Torino (Torino, Italy).
- Alberto Farina, Université de Picardie Jules Vernes, (Amiens, France).
- Antonio J. Fernández, Universidad Autónoma de Madrid (Madrid, Spain).
- Damien Galant, UMons (Mons, Belgium) and Université Polytechnique Hauts-de-France (Valenciennes, France).
- Marco Gallo, Università Cattolica del Sacro Cuore (Brescia, Italy).
- Filippo Gazzola, Politecnico di Milano (Milano, Italy).
- Senoussi Guesmia, University of the Bahamas, (Nassau, The Bahamas).

- François Hamel, Université d'Aix-Marseille (Marseille, France).
- Louis Jeanjean, Université de Bourgogne Franche-Comté (Besançon, France).
- Noureddine Lamsahel, Université du Littoral Côte d'Opale (Calais, France).
- Erwan Le Quiniou, Université de Lille (Lille, France).
- Miguel Martinez Teruel, Universidad de Granada (Granada, Spain).
- François Murat, Laboratoire Jacques-Louis Lions, Sorbonne Université (Paris, France).
- Serge Nicaise, Université Polytechnique Hauts-de-France (Valenciennes, France).
- Teresa Oitabén Santos, Université Polytechnique Hauts-de-France (Valenciennes, France) and Université de Bourgogne Franche-Comté (Besançon, France).
- Rosa Pardo, Universidad Complutense de Madrid (Madrid, Spain).
- Gaëlle Pincet Mailly, Université du Littoral Côte d'Opale (Calais, France).
- Angela Pistoia, Sapienza Università di Roma, (Roma, Italy).
- Camilla Polvara, Sapienza Università di Roma (Roma, Italy).
- Hussein Saleh, Université Polytechnique Hauts-de-France (Valenciennes, France).
- Enrico Serra, Politecnico di Torino (Torino, Italy).
- Boris Shakarov, Université de Toulouse (Toulouse, France).
- Philippe Souplet, Université Sorbonne Paris Nord (Paris, France).
- Guido Sweers, Universität zu Köln (Cologne, Germany).
- Christophe Troestler, UMons (Mons, Belgium).

4 Practical informations

Map of the campus

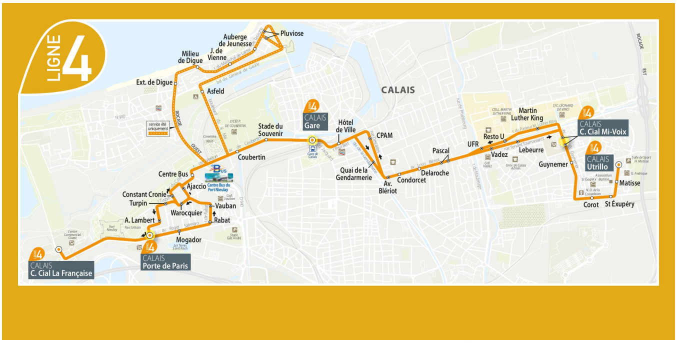
The conference will take place in Amphi C002 in Bâtiment C. You are expected to access the University Campus by entrance 1 close to Bâtiment B.

Wifi at ULCO **Eduroam** is avalaible at ULCO.

Restaurants Here is a non-exhaustive list of restaurants that we recommend in the center of Calais. We do not guarantee that they are open during the full congress. We advise you to call them and book a table.

- **Histoire ancienne**, 20 rue Royale, 62100 CALAIS, 0321341120
- **Café de Paris**, 72 rue Royale, 62100 CALAIS, 0321347684
- **La Mer**, 30 rue de la Mer, 62100 CALAIS, 0321961772
- **Crêperie Tonnerre de Brest**, 16 Place d'Armes, 62100 CALAIS, 0321969535
- **La route des épices**, 12 rue Jean de Vienne, 62100 CALAIS, 0321179653
- **Le Venezia**, 24 rue de la Mer, 62100 CALAIS, 0321978010
- **Australian's pizza**, 35 Place d'Armes, 62100 CALAIS, 0321961721
- **Marrakech**, 28 Blvd des Alliés, 62100 CALAIS, 0361310470
- **Au côte d'argent**, 1 Digue Gaston Berthe, 62100 CALAIS, 0321346807
- **Le grand bleu**, 8 rue Jean-Pierre Avron, 62100 CALAIS, 0321979798

Public transport The best way to get from downtown area to ULCO without a car is to use line 4 of Imag'in Bus (Bus stop **Vadez** or **Resto U**).



LIGNE 4

Utrillo – C. Cial Mi-Voix – Condorcet – CPAM – Gare – Coubertin – Centre bus – Warocquier – Porte de Paris – C. Cial La Française

| Arrêt non desservi

DU LUNDI AU SAMEDI - MATIN

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WAROCQUIER	6:47	7:07	7:19	7:29	7:43	7:54	8:18	8:32	8:43	8:54	9:11	9:25	9:39	9:53	10:07	10:21	10:35	10:49	11:03	11:17	11:31	11:45	11:59	12:10	12:23	12:41	12:55	13:09	13:23	13:37
PORTE DE PARIS	6:50	7:10	7:22	7:32	7:47	7:58	8:22	8:36	8:47	8:58	9:15	9:29	9:43	9:57	10:11	10:25	10:39	10:53	11:07	11:21	11:35	11:49	12:03	12:14	12:27	12:45	12:59	13:13	13:27	13:41
C. CIAL LA FRANÇAISE	6:54	7:14	7:26	7:36	7:51	8:02	8:26	8:40	8:51	9:02	9:19	9:33	9:47	10:01	10:15	10:29	10:43	10:57	11:11	11:25	11:39	11:53	12:07	12:18	12:31	12:49	13:03	13:17	13:31	13:45

DU LUNDI AU SAMEDI - APRÈS-MIDI

UTRILLO	13:24	13:41	13:55	14:09	14:23	14:37	14:51	15:05	15:19	15:33	15:47	16:01	16:15	16:29	16:43	16:57	17:11	17:25	17:39	17:53	18:07	18:21	18:35	18:49	19:03	19:12	19:24	19:39	19:50	20:03
C. CIAL MI-VOIX	13:28	13:45	13:59	14:13	14:27	14:41	14:55	15:09	15:23	15:37	15:51	16:05	16:19	16:33	16:47	17:01	17:15	17:29	17:43	17:57	18:11	18:25	18:39	18:53	19:07	19:16	19:28	19:43	19:54	20:07
CONDORCET	13:36	13:53	14:07	14:21	14:35	14:49	15:03	15:17	15:31	15:45	15:59	16:13	16:27	16:41	16:55	17:09	17:23	17:37	17:51	18:05	18:19	18:33	18:47	19:01	19:15	19:24	19:36	19:51	20:02	20:15
CPAM	13:37	13:54	14:08	14:22	14:36	14:50	15:04	15:18	15:32	15:46	16:00	16:14	16:28	16:42	16:56	17:10	17:24	17:38	17:52	18:06	18:20	18:34	18:48	19:02	19:16	19:25	19:37	19:52	20:03	20:16
GARE	13:45	13:57	14:11	14:25	14:39	14:53	15:07	15:21	15:35	15:49	16:03	16:17	16:31	16:45	16:59	17:13	17:27	17:41	17:55	18:09	18:23	18:37	18:51	19:05	19:19	19:28	19:45	19:55	20:06	20:19
COUBERTIN	13:49	14:01	14:15	14:29	14:43	14:57	15:11	15:25	15:39	15:53	16:07	16:21	16:35	16:49	17:03	17:17	17:31	17:45	17:59	18:13	18:27	18:41	18:55	19:09	19:23	19:32	19:48	19:58	20:09	20:22
CENTRE BUS	13:51	14:03	14:17	14:31	14:45	14:59	15:13	15:27	15:41	15:55	16:09	16:23	16:37	16:51	17:05	17:19	17:33	17:47	18:01	18:15	18:29	18:43	18:57	19:11	19:25	19:33	19:49	19:59	20:10	20:23
WAROCQUIER	13:53	14:05	14:19	14:33	14:47	15:01	15:15	15:29	15:43	15:57	16:11	16:25	16:39	16:53	17:07	17:21	17:35	17:49	18:03	18:17	18:31	18:45	18:59	19:13	19:27	19:35	19:51	20:01	20:12	20:25
PORTE DE PARIS	13:57	14:09	14:23	14:37	14:51	15:05	15:19	15:33	15:47	16:01	16:15	16:29	16:43	16:57	17:11	17:25	17:39	17:53	18:07	18:21	18:35	18:49	19:03	19:17	|	|	19:54	|	|	|
C. CIAL LA FRANÇAISE	14:01	14:13	14:27	14:41	14:55	15:09	15:23	15:37	15:51	16:05	16:19	16:33	16:47	17:01	17:15	17:29	17:43	17:57	18:11	18:25	18:39	18:53	19:07	19:21	|	|	19:58	|	|	|

LIGNE 4

C. Cial La Française – Porte de Paris – Warocquier – Centre bus – Coubertin – Gare – Quai gendarmerie – Condorcet – C. Cial Mi-Voix – Utrillo

| Arrêt non desservi

DU LUNDI AU SAMEDI - MATIN

C. CIAL LA FRANÇAISE	|	|	|	|	|	6:56	7:17	7:29	7:41	7:54	8:07	|	8:32	8:43	8:57	9:11	9:25	9:39	9:53	10:07	10:21	10:35	10:49	11:03	11:17	11:31	11:45	11:55	12:13	12:27	12:41
PORTE DE PARIS	|	|	|	|	|	6:59	7:20	7:32	7:44	7:57	8:10	|	8:35	8:46	9:00	9:14	9:28	9:42	9:56	10:10	10:24	10:38	10:52	11:06	11:20	11:34	11:48	11:58	12:16	12:30	12:44
WAROCQUIER	|	|	|	|	|	7:02	7:23	7:35	7:47	8:00	8:13	|	8:38	8:49	9:03	9:17	9:31	9:45	9:59	10:13	10:27	10:41	10:55	11:09	11:23	11:37	11:51	12:01	12:19	12:33	12:47
CENTRE BUS	6:00	6:18	6:30	6:39	6:50	7:04	7:25	7:37	7:49	8:02	8:15	8:27	8:40	8:51	9:05	9:19	9:33	9:47	10:01	10:15	10:29	10:43	10:57	11:11	11:25	11:39	11:53	12:03	12:21	12:35	12:49
COUBERTIN	6:02	6:20	6:32	6:41	6:52	7:06	7:27	7:39	7:51	8:04	8:17	8:29	8:42	8:53	9:07	9:21	9:35	9:49	10:03	10:17	10:31	10:45	10:59	11:13	11:27	11:41	11:55	12:08	12:23	12:37	12:51
GARE	6:06	6:24	6:36	6:45	6:56	7:10	7:35	7:43	7:55	8:08	8:21	8:33	8:46	8:57	9:11	9:25	9:39	9:53	10:07	10:21	10:35	10:49	11:03	11:17	11:31	11:45	11:59	12:15	12:27	12:41	12:56
QUAI GENDARMERIE	6:08	6:26	6:38	6:47	6:58	7:12	7:37	7:45	7:57	8:10	8:23	8:35	8:48	8:59	9:13	9:27	9:41	9:55	10:09	10:23	10:37	10:51	11:05	11:19	11:33	11:47	12:01	12:17	12:29	12:43	12:57
CONDORCET	6:10	6:28	6:40	6:49	7:00	7:14	7:39	7:47	7:59	8:12	8:25	8:37	8:50	9:01	9:15	9:29	9:43	9:57	10:11	10:25	10:39	10:53	11:07	11:21	11:35	11:49	12:03	12:19	12:31	12:45	12:59
C. CIAL MI-VOIX	6:17	6:35	6:47	6:56	7:07	7:21	7:46	7:54	8:06	8:20	8:33	8:45	8:58	9:09	9:23	9:37	9:51	10:05	10:19	10:33	10:47	11:01	11:15	11:29	11:43	11:57	12:11	12:27	12:39	12:53	13:07
UTRILLO	6:23	6:41	6:53	7:02	7:13	7:27	7:52	8:00	8:12	8:26	8:39	8:51	9:04	9:15	9:29	9:43	9:57	10:11	10:25	10:39	10:53	11:07	11:21	11:35	11:49	12:03	12:17	12:33	12:45	12:59	13:13

DU LUNDI AU SAMEDI - APRÈS-MIDI

C. CIAL LA FRANÇAISE	12:55	13:09	13:26	13:37	13:51	14:05	14:19	14:33	14:47	15:01	15:15	15:29	15:43	16:00	16:11	16:25	16:39	16:53	17:07	17:21	17:35	17:49	18:03	18:17	18:31	18:45	18:59	19:13	19:33	20:00
PORTE DE PARIS	12:58	13:12	13:29	13:40	13:54	14:08	14:22	14:36	14:50	15:04	15:18	15:32	15:46	16:03	16:14	16:28	16:42	16:56	17:10	17:24	17:38	17:52	18:06	18:20	18:34	18:48	19:02	19:16	19:36	20:03
WAROCQUIER	13:01	13:15	13:32	13:43	13:57	14:11	14:25	14:39	14:53	15:07	15:21	15:35	15:49	16:06	16:17	16:31	16:45	16:59	17:13	17:27	17:41	17:55	18:09	18:23	18:37	18:51	19:05	19:19	19:38	20:05
CENTRE BUS	13:03	13:17	13:34	13:45	13:59	14:13	14:27	14:41	14:55	15:09	15:23	15:37	15:51	16:08	16:19	16:33	16:47	17:01	17:15	17:29	17:43	17:57	18:11	18:25	18:39	18:53	19:07	19:21	19:40	20:07
COUBERTIN	13:05	13:19	13:36	13:47	14:01	14:15	14:29	14:43	14:57	15:11	15:25	15:39	15:53	16:10	16:21	16:35	16:49	17:03	17:17	17:31	17:45	17:59	18:13	18:27	18:41	18:55	19:09	19:23	19:42	|
GARE	13:09	13:23	13:45	13:51	14:05	14:19	14:33	14:47	15:01	15:15	15:29	15:43	15:57	16:14	16:25	16:39	16:53	17:07	17:21	17:35	17:49	18:03	18:17	18:31	18:45	18:59	19:13	19:27	19:45	|
QUAI GENDARMERIE	13:11	13:25	13:47	13:53	14:07	14:21	14:35	14:49	15:03	15:17	15:31	15:45	15:59	16:16	16:27	16:41	16:55	17:09	17:23	17:37	17:51	18:05	18:19	18:33	18:47	19:01	19:15	19:29	19:47	|
CONDORCET	13:13	13:27	13:49	13:55	14:09	14:23	14:37	14:51	15:05	15:19	15:33	15:47	16:01	16:18	16:29	16:43	16:57	17:11	17:25	17:39	17:53	18:07	18:21	18:35	18:49	19:03	19:17	19:31	19:49	|
C. CIAL MI-VOIX	13:21	13:35	13:57	14:03	14:17	14:31	14:45	14:59	15:13	15:27	15:41	15:55	16:09	16:26	16:37	16:51	17:05	17:19	17:33	17:47	18:01	18:15	18:29	18:43	18:57	19:10	19:24	19:38	19:56	|
UTRILLO	13:27	13:41	14:03	14:09	14:23	14:37	14:51	15:05	15:19	15:33	15:47	16:01	16:15	16:32	16:43	16:57	17:11	17:25	17:39	17:53	18:07	18:21	18:35	18:49	19:03	19:16	19:30	19:44	20:02	|

LIGNE 4

Utrillo – C. Cial Mi-Voix – Condorcet – CPAM – Gare – Coubertin – Pluviose – Extrémité de Digue – Centre bus – Warocquier – Porte de Paris – C. Cial La Française

DIMANCHE ET JOURS FÉRIÉS

UTRILLO	|	11:00	|	14:39	|	16:02	16:36	17:30	18:14	18:55	20:03
C. CIAL MI-VOIX	|	11:04	|	14:43	|	16:06	16:40	17:34	18:18	18:59	20:07
CONDORCET	|	11:12	|	14:51	|	16:14	16:48	17:42	18:26	19:07	20:15
CPAM	|	11:13	|	14:52	|	16:15	16:49	17:43	18:27	19:08	20:16
GARE	9:50	11:16	13:45	14:55	15:05	16:18	16:52	17:46	18:30	19:11	20:19
COUBERTIN	9:54	11:20	13:49	14:59	15:09	16:22	16:56	17:50	|	19:15	20:22
PLUVIOSE	|	11:27	|	|	15:16	|	17:03	|	|	|	|
EXTRÉMITÉ DE DIGUE	|	11:31	|	|	15:20	|	17:07	|	|	|	|
CENTRE BUS	9:56	11:33	13:51	15:01	15:22	16:24	17:09	17:52	|	19:17	20:23
WAROCQUIER	9:58	11:35	13:53	15:03	15:24	16:26	17:11	17:54	|	19:19	20:25
PORTE DE PARIS	10:02	11:39	13:57	15:07	15:28	16:30	17:15	17:58	|	19:23	|
C. CIAL LA FRANÇAISE	10:06	11:43	14:01	15:11	15:32	16:34	17:19	18:02	|	19:27	|

C. Cial La Française – Porte de Paris – Warocquier – Centre bus – Extrémité de Digue – Pluviose – Coubertin – Gare – Quai gendarmerie – Condorcet – C. Cial Mi-Voix – Utrillo

DIMANCHE ET JOURS FÉRIÉS

C. CIAL LA FRANÇAISE	10:08	11:45	14:02	15:16	15:40	16:44	17:24	18:12	19:30
PORTE DE PARIS	10:11	11:48	14:05	15:19	15:43	16:47	17:27	18:15	19:33
WAROCQUIER	10:14	11:51	14:08	15:22	15:46	16:50	17:30	18:18	19:35
CENTRE BUS	10:16	11:53	14:10	15:24	15:48	16:52	17:32	18:20	19:37
EXTRÉMITÉ DE DIGUE	10:19	|	|	|	15:52	|	17:36	|	|
PLUVIOSE	10:22	|	|	|	15:56	|	17:40	|	|
COUBERTIN	10:28	|	14:12	15:26	16:03	16:54	17:47	18:22	19:39
STADE DU SOUVENIR	10:29	|	14:13	15:27	16:04	16:55	17:48	18:23	19:40
GARE	10:32	|	14:16	15:30	16:07	16:58	17:51	18:26	19:45
QUAI GENDARMERIE	10:34	|	14:18	15:32	16:09	17:00	17:53	18:28	19:47
CONDORCET	10:36	|	14:20	15:34	16:11	17:02	17:55	18:30	19:49
C. CIAL MI-VOIX	10:44	|	14:28	15:42	16:19	17:10	18:03	18:38	19:56
UTRILLO	10:50	|	14:34	15:48	16:25	17:16	18:09	18:44	20:02

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